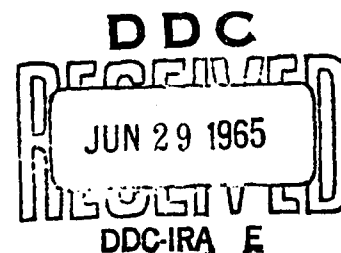UNCLASSIFIED

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HYDRONAUTICS, incorporated
research in hydrodynamics

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TECHNICAL REPORT 231-7

ON THE SURFACE WAVE PATTERN OF
SUBMERGED BODIES
STARTED FROM REST

By

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April 1965

Prepared Under

HYDRONAUTICS, Incorporated

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NOTATION

f	depth of submergence measured from the centerline of the body
g	gravitational acceleration
M	strength of point source
$\vec{q}$	velocity vector
R	radial distance from source or sink
t	time
U	forward speed
x,y,z	rectangular cartesian coordinates
φ	velocity potential
ζ	wave height
ζ_r	regular wave height
ζ_ℓ	local disturbance
ζ_s	steady wave height
ζ_1	unsteady disturbance
$\overline{\zeta}_s, \overline{\zeta}_1$	amplitude of the steady wave and unsteady disturbance, respectively

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ABSTRACT

The initial surface wave pattern due to a submerged body
started from rest with uniform speed in a perfect fluid is
studied. The mean surface elevations may, in general, be sep-
arated into three parts: (i) A steady local disturbance which
travels both upstream and downstream and diminishes as the dis-
tance from the body increases, (ii) a group of steady regular
waves which travel downstream from the origin $x = x - x_o = 0$ to
$x = 1/2$ Ut with group velocity $1/2$ U, and (iii) a time-dependent
cylindrical disturbance which travels both upstream and down-
stream and   .nishes rapidly as time increases. For suffi-
ciently long .me t, the general expression for the unsteady dis-
turbance can be obtained by the method of stationary phase. It
is found that for a given finite value of t, the effect of the
unsteady disturbance becomes more serious as the downstream dis-
tance from the body increases. It is also shown that the ini-
tial unsteady effect is, at lower Froude numbers, more persist-
ent and therefore takes a longer time to subside. A comparison
of the predicted centerline wave profile due to a submerged
9 feet long, 7 to 1, Rankine ovoid started from rest with DTMB
data has been made; the results are in very good agreement.

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INTRODUCTION

The problem of submerged bodies or ships in accelerated motion is primarily of interest in connection with towing-tank experiments. No matter how quickly the final desired speed is attained by the model, the question always arises as to how long it takes before the effect of the starting conditions becomes inappreciable. This question has been discussed by Havelock (1), (2), Maruo (3) and Wehausen (4) in dealing with the wave resistance of submerged bodies or of thin ships. The effect of the initial acceleration upon the wave pattern of submerged bodies has received, however, very little attention. It is the purpose of this report to investigate the surface wave pattern of simple submerged bodies started from rest. The result is of practical importance in connection with the experimental determination in towing tanks of the surface waves made by submarine models. The theoretical results for the particular case of a Rankine body are compared with experiments conducted at the David Taylor Model Basin.

GENERAL FORMULATION

The governing equation for submerged bodies moving through an incompressible, inviscid fluid in irrotational motion may be shown to be

$$\nabla^2 \varphi = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0 \qquad [1]$$

where

x,y,z   are the cartesian coordinates and the origin is taken on the mean free surface with O-z vertically upwards,

φ is the perturbation velocity potential and is assumed to be small.

The perturbed fluid velocity $\vec{q}(u,v,w)$ is given by

$$\vec{q} = -\nabla\varphi = -\text{grad } \varphi \qquad [2]$$

The linearized boundary conditions at the free surface may be expressed as

$$\left. \begin{array}{ll} \dfrac{\partial\varphi}{\partial t} - g\zeta = \text{const.} & \text{(dynamical condition)} \\[3mm] \dfrac{\partial\varphi}{\partial z} + \dfrac{\partial\zeta}{\partial t} = 0 & \text{(kinematic condition)} \end{array} \right\} \text{at } z = 0 \qquad [3]$$

where ζ is the free surface elevation,

g the gravitational acceleration, and

t the time variable.

At the rigid submerged body surface it is evident that the condition

$$\frac{\partial(\varphi + Ux)}{\partial n} = 0 \qquad [4]$$

must hold, in which $\dfrac{\partial}{\partial n}$ represents differentiation along the normal to the boundary surface and where U is the free stream velocity.

Boundary condition [4] can generally be satisfied by distributing appropriate singularities inside or on the surface of the body. In order to satisfy boundary conditions [3] and [4] simultaneously, a complementary function, in addition to the prescribed singularities, is needed. It can be shown, Reference 3, that the total velocity potential must have the form

$$\varphi = \varphi_o + \varphi_1$$

$$= \varphi_o - \overline{\varphi}_o + \frac{1}{\pi} \int^{t} d\tau \int_{-\pi}^{\pi} d\theta \int_{o}^{\infty} G(\varkappa,\theta,\tau)\, e^{\varkappa(z-i\tilde{\omega})} \sin\left\{\sqrt{g\varkappa}(t-\tau)\right\} \sqrt{g\varkappa}\; d\varkappa \quad [5]$$

where

φ_o is the velocity potential due to the prescribed singularities and depends mainly on the body shape

$\overline{\varphi}_o$ is the image of φ_o

$$G(\varkappa,\theta,\tau) = -\frac{1}{4\pi} \int_{S'} \left(\frac{\partial \varphi_o}{\partial n} - \varphi_o \frac{\partial}{\partial n}\right) e^{\varkappa(z+i\tilde{\omega})}\; dS$$

S' is an arbitrary surface enclosing the singularities

$\tilde{\omega}$ $= x \cos \theta + y \sin \theta .$

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Consider now the special case of a Rankine body (ovoid) submerged in water at a given depth, as shown in Figure 1, suddenly started from rest with a given velocity U and maintaining that speed. For a first order approximation, the fluid motion may be taken to be that due to a point source and a point sink of strength $|M|$ distributed at the points $(0,0,-f)$ and $(C, \ell, -f)$ respectively. The velocity potential φ_0 due to an impulsive point source or sink at (x_n, y_n, z_n) is given by

$$\varphi_0 = \frac{M_n}{R_n} \qquad [6]$$

where

$$M_n(t) = 0 \qquad\qquad t \leq 0$$

$$M_n(t) = M_n = \text{constant} \qquad t > 0$$

$$R_n = \sqrt{(x-x_n)^2 + (y-y_n)^2 + (z-z_n)^2}$$

The function G of Equation [5] in this case is simply

$$G = M_n \, e^{\varkappa[z_n + i(x_n \cos \theta + y_n \sin \theta)]} \qquad [7]$$

Substituting Equation [7] into Equation [5] gives

$$\varphi = \frac{M_n}{R_n} - \frac{M_n}{\overline{R}_n} + \frac{1}{\pi} \int_0^t M_n(\tau)d\tau \int_{-\pi}^{\pi} d\theta \int_0^{\infty} e^{\varkappa(z+z_n)} \cos \varkappa \tilde{\omega}_n' \sin\left\{\sqrt{g\varkappa}(t-\tau)\right\} \sqrt{g\varkappa} \, d\varkappa$$

[8]

where

$$\overline{R}_n = \sqrt{(x-x_n)^2 + (y-y_n)^2 + (z+z_n)^2}$$

$$\tilde{\omega}_n' = (x-x_n) \cos \theta + (y-y_n) \sin \theta$$

The total velocity potential for a Rankine ovoid started impulsively from rest with uniform velocity can then be shown to be

$$\varphi = \varphi_{source} + \varphi_{sink}$$

$$= M\left[\frac{1}{\sqrt{x^2 +y^2 +(z+f)^2}} - \frac{1}{\sqrt{x^2 +y^2 +(z-f)^2}} + \frac{1}{\sqrt{(x+\ell)^2 +y^2 +(z-f)^2}} - \frac{1}{\sqrt{(x+\ell)^2+y^2+(z+f)^2}}\right]$$

$$+ \frac{1}{\pi} \int_0^t M(\tau)d\tau \int_{-\pi}^{\pi} d\theta \int_0^{\infty} e^{-\varkappa f+\varkappa z}\left[\cos \varkappa\tilde{\omega}_1' - \cos \varkappa\tilde{\omega}_2'\right]\sin\left\{\sqrt{g\varkappa}(t-\tau)\right\} \sqrt{g\varkappa} \, d\varkappa$$

[9]

with

$$\tilde{\omega}_1' = x \cos \theta + y \sin \theta \qquad\qquad \tilde{\omega}_2' = (x+\ell)\cos \theta + y \sin \theta$$

$$\ell = \text{distance between the point source and sink}$$

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SURFACE WAVE PROFILES

It is seen from Equation [3] that the mean free surface eleva-
tions can be expressed as

$$\zeta \approx \frac{1}{g}\frac{\partial \varphi}{\partial t}\bigg|_{z=0}$$

$$= \frac{1}{g}\frac{\partial}{\partial t}(\varphi_0 - \bar{\varphi}_0) + \frac{1}{\pi}\int_0^t d\tau \int_{-\pi}^{\pi} d\theta \int_0^{\infty} \varkappa G(\varkappa,\theta,\tau) e^{\varkappa(z-i\tilde{\omega})}\cos\left\{\sqrt{g\varkappa}(t-\tau)\right\} d\varkappa \bigg|_{\text{at } z=0}$$

[10]

Consider first the case of an impulsive point source started from
rest with velocity U which is maintained constant; the mean sur-
face wave height can then be expressed as

$$\zeta = \frac{1}{\pi} R\ell \int_0^t M(\tau)\, d\tau \int_{-\pi}^{\pi} d\theta \int_0^{\infty} \varkappa\, e^{-\varkappa f}\, e^{i\varkappa\tilde{\omega}}\cos\sqrt{g\varkappa}(t-\tau)\, d\varkappa \qquad [11]$$

with

$$\tilde{\omega} = \left[(x-x_0) + U(t-\tau)\right]\cos\theta + y\sin\theta$$

$$R\ell \qquad \text{denotes the real part of}$$

Integrating with respect to τ, Equation [11] becomes

$$\zeta = \frac{1}{2\pi} \left(\frac{M}{U}\right) R\ell \int_{-\pi}^{\pi} d\theta \int_{0}^{\infty} \left\{ 1 \frac{2\varkappa \sec \theta}{\varkappa - \frac{g}{U^2} \sec^3 \theta} e^{-\varkappa f + i\varkappa[(x-x_0)\cos\theta + y\sin\theta]} \right.$$

$$\left. + \frac{\varkappa}{1} e^{-\varkappa f + i\varkappa[(x-x_0)\cos\theta + y\sin\theta]} \left[\frac{e^{i(\varkappa U\cos\theta - \sqrt{g\varkappa})t}}{\varkappa\cos\theta - \sqrt{\frac{g}{U^2}\varkappa}} + \frac{e^{i(\varkappa U\cos\theta + \sqrt{g\varkappa})t}}{\varkappa\cos\theta + \sqrt{\frac{g}{U^2}\varkappa}} \right] \right\} d\varkappa d\theta$$

$$= \zeta_s + \zeta_i \qquad\qquad [12]$$

The limiting value of the regular wave ζ_r as t becomes infinite may
be derived from the principle values of the integrals in Equa-
tion [12]. The steady local disturbance ζ_ℓ arises from the first
integral in Equation [12]. The steady surface wave pattern $(\zeta_s = \zeta_r + \zeta_\ell)$
due to a point source has been discussed in detail in a previous re-
port by Yim (5). The primary interest here is to evaluate approxi-
mately the effect the initial time-dependent disturbances have on
the general wave profiles.

For sufficiently large positive values of t, the surface wave
heights due to the initial disturbances may be evaluated from the
following double integral by applying the method of stationary phase
twice in succession:

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$$\zeta_1 = \frac{1}{\pi}\frac{M}{U}\,R\ell\left\{\int_{-\pi/2}^{\pi/2}d\theta\int_0^\infty \frac{\varkappa\,e^{-\varkappa f}}{1}\left[\frac{e^{i(\varkappa\tilde{w}_1 - \sqrt{g\varkappa}\,t)}}{\varkappa\cos\theta - \sqrt{\frac{g}{U^2}\varkappa}} + \frac{e^{i(\varkappa\tilde{w}_1 + \sqrt{g\varkappa}\,t)}}{\varkappa\cos\theta + \sqrt{\frac{g}{U^2}\varkappa}}\right]d\varkappa\right\}$$

[13]

with

$$\tilde{w}_1 = [(x-x_0) + Ut]\cos\theta + y\sin\theta = R_1\cos(\theta-\delta) > 0$$

$$R_1 = \sqrt{(x - x_0 + Ut)^2 + y^2}$$

$$\delta_1 = \tan^{-1}\left(\frac{y}{x - x_0 + Ut}\right)$$

Evaluating fir. with respect to $\varkappa$, Equation [13] has the form

$$\zeta_1 \simeq \frac{1}{\sqrt{\pi}}\frac{M}{U}R\ell\left\{\frac{1}{1}\int_{-\pi/2}^{+\pi/2}\frac{\exp\left[\frac{gt^2 f}{4R_1{}^2\cos^2(\theta-\delta_1)} - i\left(\frac{gt^2}{4R_1\cos(\theta-\delta_1)} - \frac{\pi}{4}\right)\right]d\theta}{\sqrt{\frac{R_1{}^3\cos^3(\theta-\delta_1)}{gt^2}}\left(\cos\theta - \frac{2R_1\cos(\theta-\delta_1)}{Ut}\right)}\right\}$$

[14]

The initial wave heights ζ_1 for sufficiently large time t, may then be shown to be

$$\zeta_1 \simeq \frac{M}{U} R \ell \left\{ \frac{1}{1} \frac{\exp\left[\frac{-gt^2}{4R_1^2} f - i \frac{gt^2}{4R_1}\right]}{\left(\sqrt{\frac{1}{8}\left|\frac{-gt^2}{R_1}\right|}\right)\sqrt{\frac{R_1^3}{gt^2}}\left(\cos \delta_1 - \frac{2R_1}{Ut}\right)} \right\}$$

$$= -2\sqrt{2}\, \frac{M}{U} \frac{e^{\frac{-gt^2}{4R_1^2} f}}{R_1\left(\cos \delta_1 - \frac{2R_1}{Ut}\right)} \sin \frac{gt^2}{4R_1} \qquad [15]$$

This formula however is not valid in the neighborhood of $R_1 = 0$ and $R_1 = Ut/2 \cos \delta_1$ since in these regions the non-circular part of the integrand in Equation [14] varies rather rapidly and the method of stationary phase therefore fails to apply. The correct evaluation of ζ_1 around these points, which is generally very tedious and has little bearing on the outcome, will not be pursued further.

The mean water free surface elevation due to a Rankine ovoid started impulsively from rest with constant velocity U can be obtained by superposing the values due to a point source and sink with distance ℓ apart. The resulting disturbance may be separated into three parts as Havelock did in (1) for the two-dimensional case (see Figure 2).

(1) A steady local disturbance (ζ_ℓ) which traveled both upstream and downstream and diminishes as the distance from the body increases.

(ii) A group of steady regular waves (ζ_r) which travel downstream from the origin $x-x_o=x=0$ only to $x=-1/2\ Ut$ with group velocity $1/2\ U$, and behave like $x^{-\frac{1}{2}}$ far downstream $(1 \ll x/\lambda < Ut/2\lambda)$.

(iii) A time-dependent cylindrical disturbance (ζ_1) which travels both upstream and downstream and diminishes rapidly as time increases.

The first two parts have been studied in (5). The last part, for sufficiently long time t, is simply

$$\zeta_1 \approx -2\sqrt{2}\ \frac{M}{U}\left\{\frac{e^{\frac{-gt^2}{4R_1^3}f}\sin\frac{gt^2}{4R_1}}{R_1\left(\cos\delta_1-\frac{2R_1}{Ut}\right)}-\frac{e^{\frac{-gt^2}{4R_2^3}f}\sin\frac{gt^2}{4R_2}}{R_2\left(\cos\delta_2-\frac{2R_1}{Ut}\right)}\right\} \qquad [16]$$

where

$$R_1=\sqrt{(x+Ut)^2+y^2}\ ,\qquad R_2=\sqrt{(x+\ell+Ut)^2+y^2}$$

$$\delta_1=\tan^{-1}\left(\frac{y}{x+Ut}\right)\ ,\qquad \delta_2=\tan^{-1}\left(\frac{y}{x+\ell+Ut}\right)$$

respectively.

In order to assess quantitatively the initial unsteady disturbance on the overall wave motion, simple numerical calculations are made in the following section.

NUMERICAL RESULTS AND DISCUSSION

For simplicity only the waves on the centerline of the mean water free surface due to a Rankine ovoid started impulsively from rest are to be discussed in detail here. From Equation [16], taking $y = 0$, the unsteady cylindrical wave pattern ζ_1 for sufficiently long time takes the form

$$\zeta_1 = +2\sqrt{2}\left(\frac{M}{U}\right)\left(\frac{1}{Ut}\right)\left\{\frac{e^{\dfrac{-gf}{4U^2\left(1+\frac{x}{Ut}\right)^2}}\sin\left[\dfrac{gt}{4U\left(1+\frac{x}{Ut}\right)}\right]}{\left(1+\frac{x}{Ut}\right)\left(1+\frac{2x}{Ut}\right)} - \frac{e^{\dfrac{-gf}{4U^2\left(1+\frac{x+\ell}{Ut}\right)^2}}\sin\left[\dfrac{gt}{4U\left(1+\frac{x+\ell}{Ut}\right)}\right]}{\left(1+\frac{x+\ell}{Ut}\right)\left[1+\frac{2(x+\ell)}{Ut}\right]}\right\}$$

$$= \bar{\zeta}_1\left\{\sin\left[\frac{gt}{4U\left(1+\frac{x}{Ut}\right)}\right] - A_1\sin\left[\frac{gt}{4U\left(1+\frac{x+\ell}{Ut}\right)}\right]\right\} \qquad [17]$$

where

$$\bar{\zeta}_1 = 2\sqrt{2}\left(\frac{M}{U}\right)\left(\frac{1}{Ut}\right)\frac{e^{-\dfrac{1}{4F^2\left(1+\frac{x}{Ut}\right)^2}}}{\left(1+\frac{x}{Ut}\right)\left(1+\frac{2x}{Ut}\right)}$$ is defined as the amplitude of the cylindrical unsteady disturbance,

$$A_1 = \frac{\left(1 + \frac{x}{Ut}\right)\left(1 + \frac{2x}{Ut}\right)}{\left(1 + \frac{x+\ell}{Ut}\right)\left[1 + \frac{2(x+\ell)}{Ut}\right]} \exp\left\{-\frac{1}{4F^2}\left[\frac{1}{\left(1 + \frac{x+\ell}{Ut}\right)^2} - \frac{1}{\left(1 + \frac{x}{Ut}\right)^2}\right]\right\} \text{ is,}$$

in general, of $O(1)$,

$F = \dfrac{U}{\sqrt{gf}}$ is the Froude number based on depth.

It can be seen that the amplitude of the unsteady cylindrical wave, $\overline{\zeta}_1$, for a fixed value of x, diminishes rapidly like $1/t$ with increasing t. At a given time t, the value of $\overline{\zeta}_1$ decreases upstream but increases downstream as the distance from the body increases. It is, therefore, to be expected, that the effect of the unsteady cylindrical disturbance on the general steady wave pattern would be more serious far downstream. To illustrate, it is assumed that the distance from the body downstream is sufficiently large so that the steady local disturbance is negligibly small and the regular wave can be evaluated by the method of stationary phase, i.e.,

$$\zeta_s = \zeta_\ell + \zeta_s \approx \zeta_s$$

$$\approx 4\frac{M}{U}\left(\frac{2\pi g}{U^2}\right)^{\frac{1}{2}} e^{-\frac{1}{F^2}}\left[\frac{1}{|x|^{\frac{1}{2}}}\cos\left(\frac{g|x|}{U^2}+\frac{\pi}{4}\right) - \frac{1}{|x+\ell|^{\frac{1}{2}}}\cos\left(\frac{g|x+\ell|}{U^2}+\frac{\pi}{4}\right)\right]$$

$$= \overline{\zeta}_s\left[\cos\left(\frac{g|x|}{U^2}+\frac{\pi}{4}\right) - A_s\cos\left(\frac{g|x+\ell|}{U^2}+\frac{\pi}{4}\right)\right] \qquad [18]$$

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where

$$\zeta_s = \pi \frac{M}{U} \left|\frac{1}{\lambda x}\right|^{\frac{1}{2}} e^{-\frac{1}{F^2}}$$ is the amplitude of the steady wave profile far downstream $(1 << x/\lambda < Ut/2\lambda)$

$$A_s = \left|\frac{x}{x+\ell}\right|^{\frac{1}{2}} \sim O(1)$$

$$\lambda = \frac{2\pi U^2}{g}$$ is the centerline wave length

From Equations [17] and [18], the amplitude ratio of the unsteady disturbance and the steady wave is found to be

$$\frac{\bar{\zeta}_1}{\zeta_s} = \frac{1}{2\sqrt{2}\pi} \frac{|\lambda x|^{\frac{1}{2}}}{Ut} \frac{e^{-\frac{1}{F^2}\left[\frac{1}{4\left(1+\frac{x}{Ut}\right)^2} - 1\right]}}{\left(1+\frac{x}{Ut}\right)\left(1+\frac{2x}{Ut}\right)}$$

$$= \frac{1}{2\sqrt{2}\pi} \left|\frac{1}{x\lambda}\right|^{\frac{1}{2}} \left|\frac{x}{Ut}\right| \frac{e^{-\frac{1}{F^2}\left[\frac{1}{4\left(1+\frac{x}{Ut}\right)^2} - 1\right]}}{\left(1+\frac{x}{Ut}\right)\left(1+\frac{2x}{Ut}\right)}$$ [19]

The non-dimensional value of $\bar{\zeta}_1/\zeta_s$, at $x/\lambda = 4$, versus Ut/λ for various Froude numbers F is shown in Figure 3. It can be seen that the value of $\bar{\zeta}_1$ diminishes very rapidly with increase in t.

In Figure 4 the amplitude ratio $\bar{\zeta}_1/\bar{\zeta}_s$, at $Ut/\lambda = 10$, is plotted against x/λ for various F. It is evident that the effect of the unsteady disturbance on the steady wave profile becomes more prominent (since the value of $\bar{\zeta}_1/\bar{\zeta}_s$ gets bigger) as the wave moves further downstream. It is also of interest to note from Figures 3 and 4, that the value of $\bar{\zeta}_1/\bar{\zeta}_s$ at a given x and t increases with decreasing F. This would seem to suggest the fact that the unsteady disturbance takes a much longer time to subside at the lower Froude numbers.

A preliminary experimental study of the Kelvin wake produced by submerged bodies has been made at the David Taylor Model Basin, (6). In those tests, a 9 feet long, 7 to 1, Rankine ovoid was towed at several submergence depths, and wave height measurements were made at various points in the basin. In Figure 5 the center-line wave pattern in a test run, at $U = 10$ ft/sec, $t = 13.22$ sec., and $F = 1.016$, together with the corresponding calculated values are shown. Since at distances very far downstream (> 1/2 Ut) the wave profile is entirely contributed by the unsteady disturbance as discussed in the previous section and is of no practical interest, the comparison between the theoretical calculations and experimental data is, therefore, made only from $x = 0$ (near the nose of the ovoid) extending downstream to the neighborhood of 1/2 Ut. It is seen that the theory and experiment are in quite good agreement. The value of F, for the particular case studied, is rather large and the effect of unsteady disturbance on the steady surface wave profile is, in general, small as can be seen from Figure 3 or 4; the measured and the steady surface profiles

are quite similar. At lower Froude numbers, say F = 1/2, however, the unsteady effect is considerably larger; it is to be expected then that the true steady surface wave pattern may have quite a departure from the measured values in this latter case.

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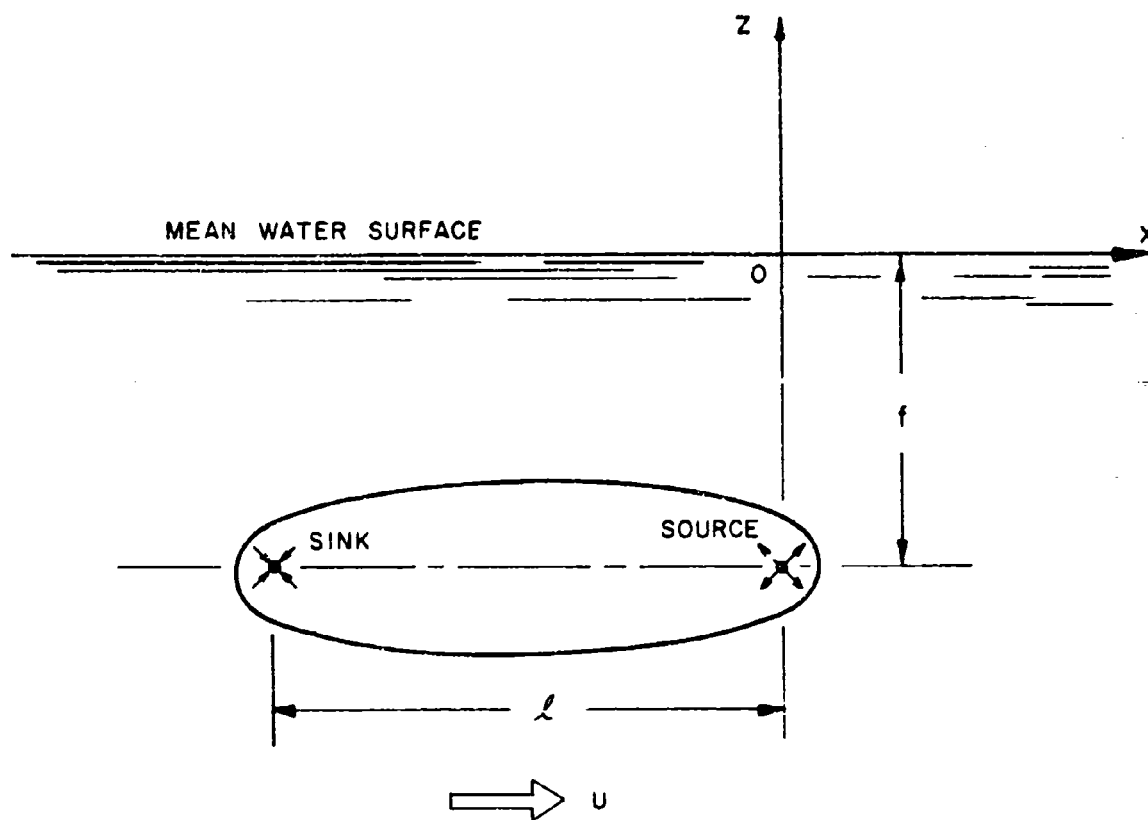6. "Preliminary Kelvin Wake Study," DTMB Test Report 039-H-01, December 1964
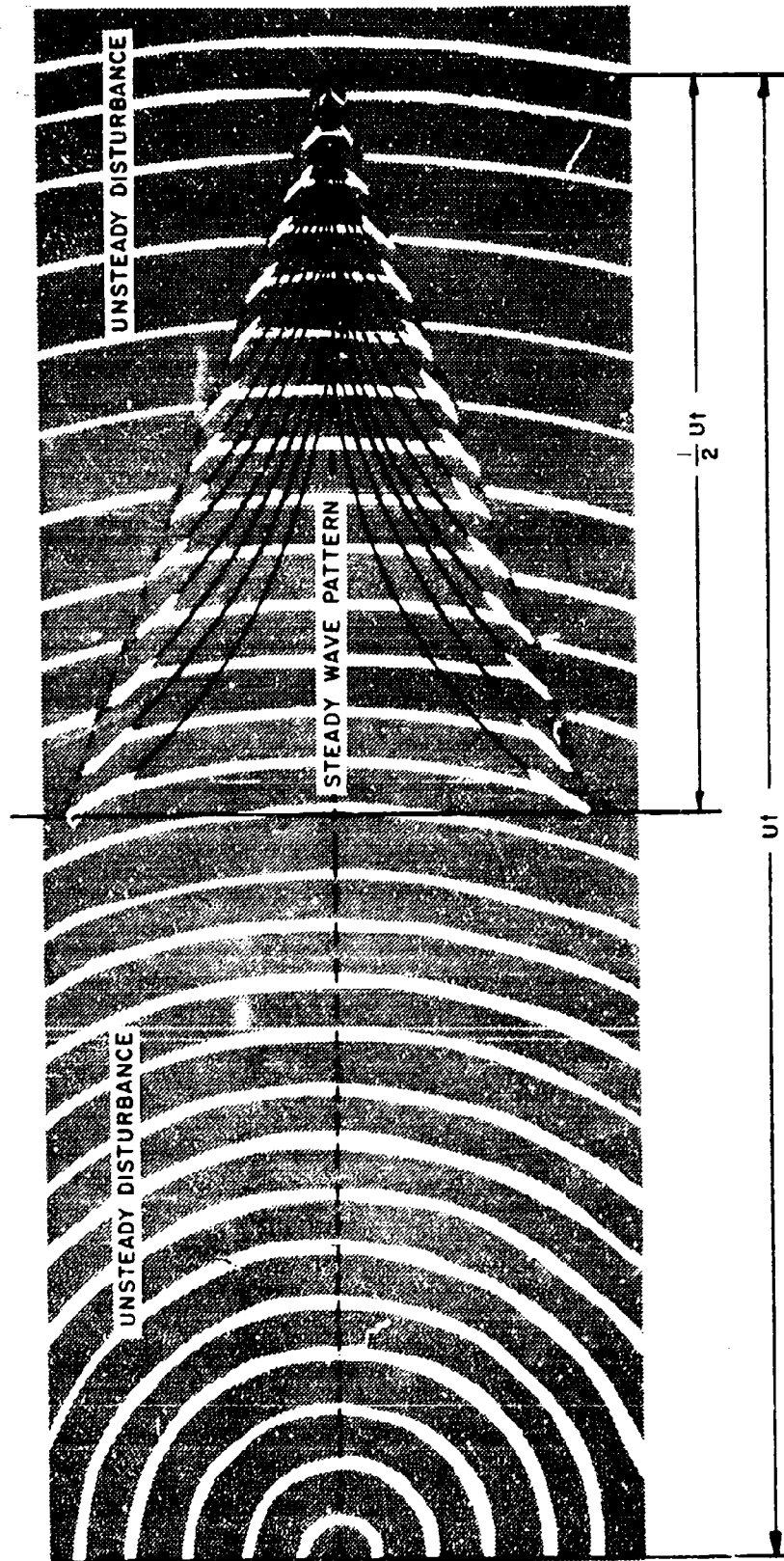
FIGURE I - DEFINITION-SKETCH

FIGURE 2—SURFACE WAVE PATTERN OF A SUBMERGED BODY STARTED IMPULSIVELY FROM REST
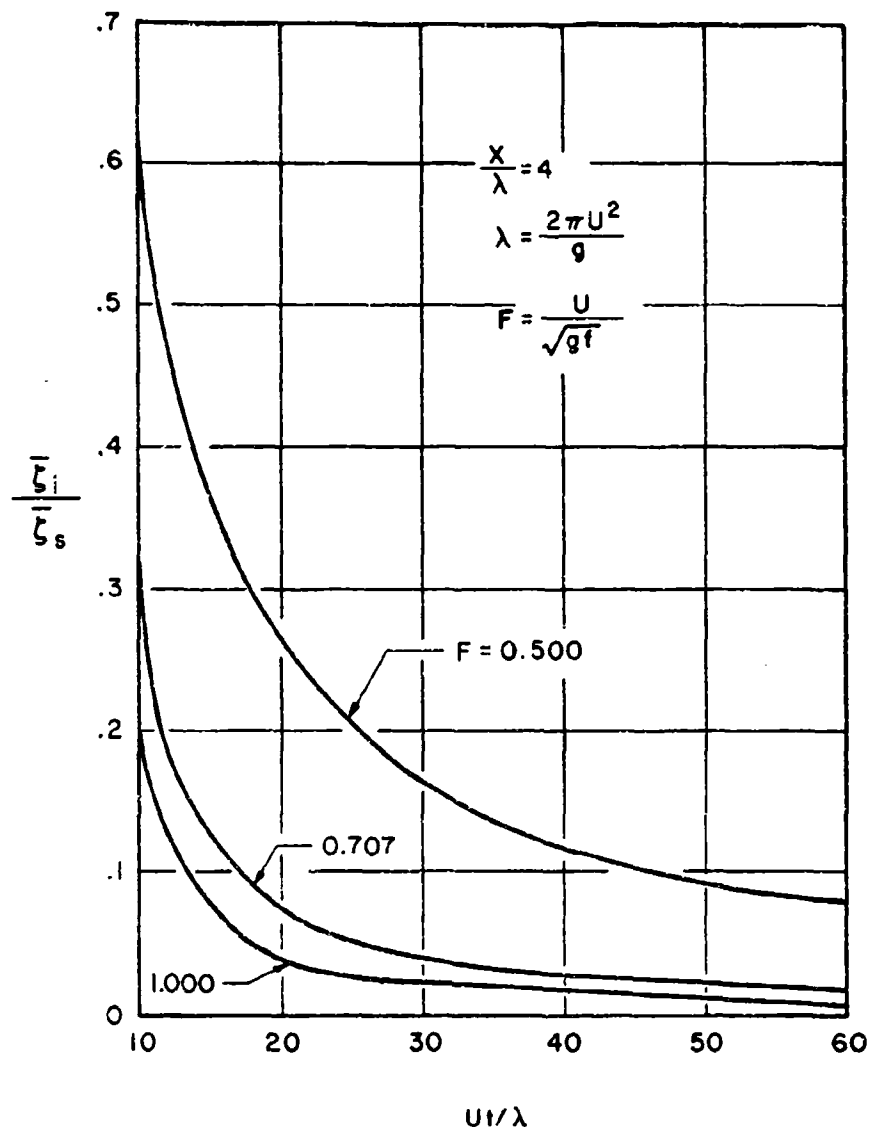
FIGURE 3 - AMPLITUDE OF THE INITIAL UNSTEADY DISTURBANCE
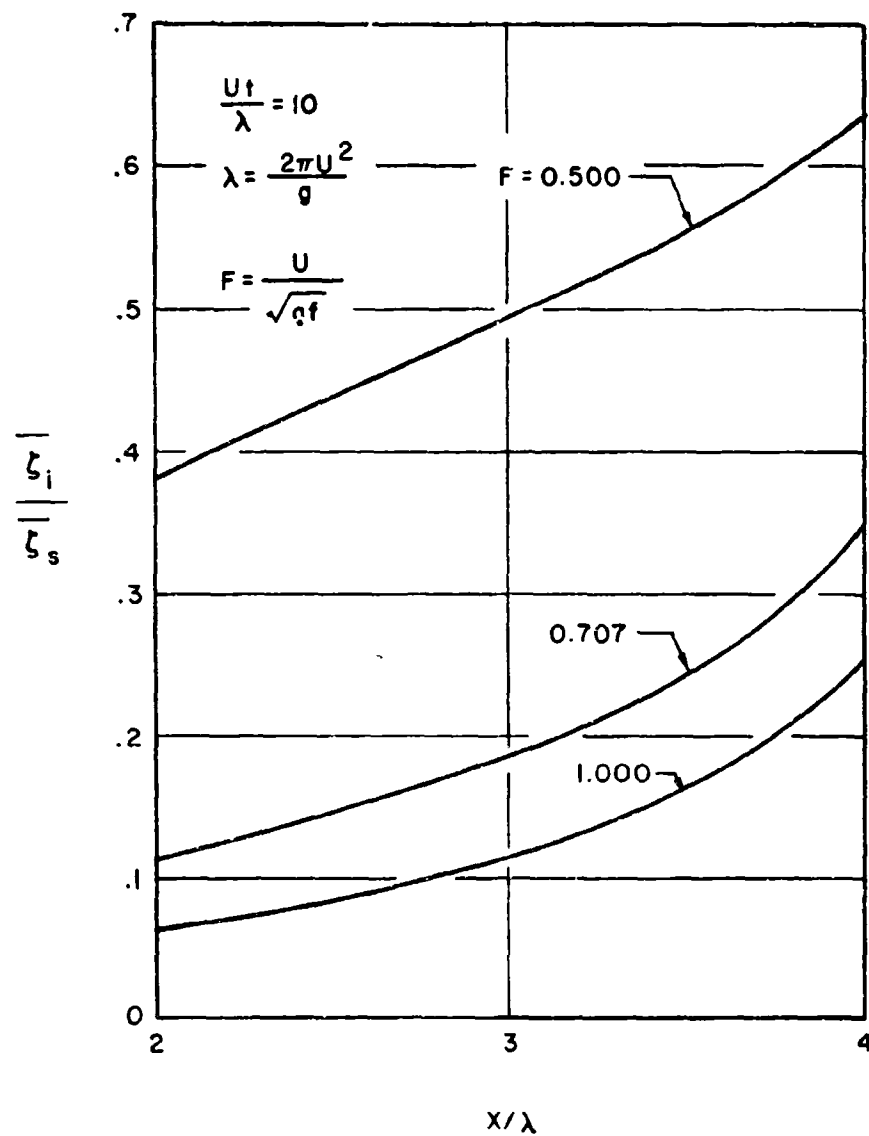WITH t AS A VARYING PARAMETER

FIGURE 4 - AMPLITUDE OF THE INITIAL UNSTEADY DISTURBANCE
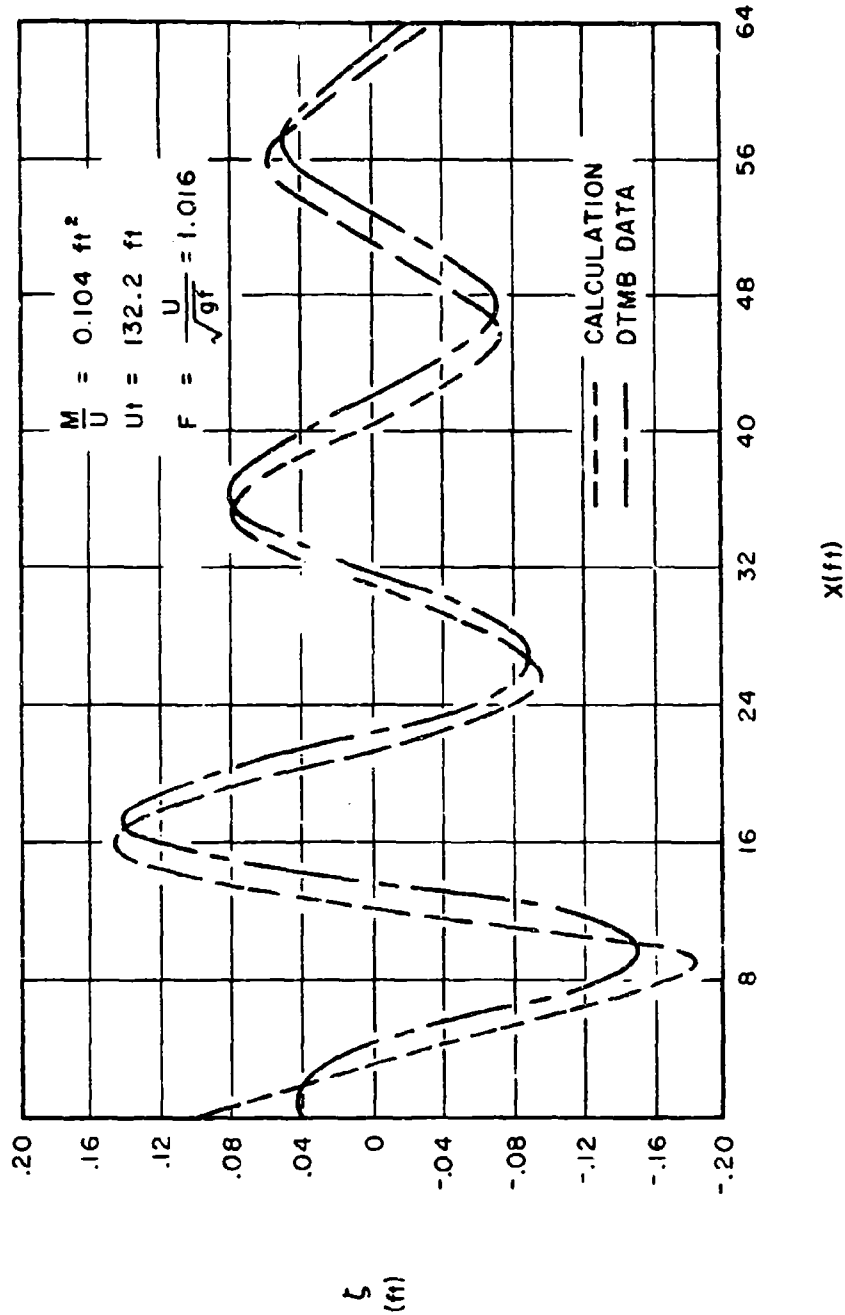WITH X AS A VARYING PARAMETER

FIGURE 5 - COMPARISON OF THEORETICAL AND EXPERIMENTAL CENTERLINE WAVE PROFILES DUE TO A SUBMERGED 9 FEET LONG, 7 TO 1, RANKINE OVOID STARTED FROM REST

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Office of Naval Research Department of the Navy

13. ABSTRACT

The initial surface wave pattern due to a submerged body started from rest with uniform speed in a perfect fluid is studied. The mean surface elevations may, in general, be separated into three parts: (i) A steady local disturbance which travels both upstream and downstream and diminishes as the distance from the body increases, (ii) a group of steady regular waves which travel downstream from the origin $x=0$ to $x=1/2$ Ut with group velocity $1/2$ U, and (iii) a time-dependent cylindrical disturbance which travels both upstream and downstream and diminishes rapidly as time increases. For sufficiently long time t, the general expression for the unsteady disturbance can be obtained by the method of stationary phase. It is found that for a given finite value of t, the effect of the unsteady disturbance becomes more serious as the downstream distance from the body increases. It is also shown that the initial unsteady effect is, at lower Froude numbers, more persistent and therefore takes a longer time to subside. A comparison of the predicted centerline wave profile due to a submerged 9 feet long, 7 to 1, Rankine ovoid started from rest with DTMB data has been made; the results are in very good agreement.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
1. Submerged body						
2. Velocity potential						
3. Impulsive motion						
4. Surface waves						
5. Unsteady disturbance						

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